Spring 2012

EE 330

ENGINEERING ELECTROMAGNETICS

HW 11: *Due Friday 13 April*
8.2, 8.4, 8.10, 8.16, 8.20, 8.27, 8.29, 8.35, 8.39, 8.41, 8.47

Problem 8.2 A plane wave traveling in medium 1 with $\varepsilon_{r1} = 2.25$ is normally incident upon medium 2 with $\varepsilon_{r2} = 4$. Both media are made of nonmagnetic, non-conducting materials. If the electric field of the incident wave is given by

$$\mathbf{E}^i = \hat{\mathbf{y}} 8\cos(6\pi \times 10^9 t - 30\pi x) \quad \text{(V/m)}.$$

(a) Obtain time-domain expressions for the electric and magnetic fields in each of the two media.

(b) Determine the average power densities of the incident, reflected and transmitted waves.

Solution:

(a)

$$\mathbf{E}^i = \hat{\mathbf{y}} 8\cos(6\pi \times 10^9 t - 30\pi x) \quad \text{(V/m)},$$

$$\eta_1 = \frac{\eta_0}{\sqrt{\varepsilon_{r_1}}} = \frac{\eta_0}{\sqrt{2.25}} = \frac{\eta_0}{1.5} = \frac{377}{1.5} = 251.33\ \Omega,$$

$$\eta_2 = \frac{\eta_0}{\sqrt{\varepsilon_{r_2}}} = \frac{\eta_0}{\sqrt{4}} = \frac{377}{2} = 188.5\ \Omega,$$

$$\Gamma = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1} = \frac{1/2 - 1/1.5}{1/2 + 1/1.5} = -0.143,$$

$$\tau = 1 + \Gamma = 1 - 0.143 = 0.857,$$

$$\mathbf{E}^r = \Gamma \mathbf{E}^i = -1.14\hat{\mathbf{y}}\cos(6\pi \times 10^9 t + 30\pi x) \quad \text{(V/m)}.$$

Note that the coefficient of x is positive, denoting the fact that $\mathbf{E}^r$ belongs to a wave traveling in $-x$-direction.

$$\mathbf{E}_1 = \mathbf{E}^i + \mathbf{E}^r = \hat{\mathbf{y}}\left[8\cos(6\pi \times 10^9 t - 30\pi x) - 1.14\cos(6\pi \times 10^9 t + 30\pi x)\right] \quad \text{(A/m)},$$

$$\mathbf{H}^i = \hat{\mathbf{z}}\frac{8}{\eta_1}\cos(6\pi \times 10^9 t - 30\pi x) = \hat{\mathbf{z}} 31.83\cos(6\pi \times 10^9 t - 30\pi x) \quad \text{(mA/m)},$$

$$\mathbf{H}^r = \hat{\mathbf{z}}\frac{1.14}{\eta_1}\cos(6\pi \times 10^9 t + 30\pi x) = \hat{\mathbf{z}} 4.54\cos(6\pi \times 10^9 t + 30\pi x) \quad \text{(mA/m)},$$

$$\begin{aligned}\mathbf{H}_1 &= \mathbf{H}^i + \mathbf{H}^r \\ &= \hat{\mathbf{z}}\left[31.83\cos(6\pi \times 10^9 t - 30\pi x) + 4.54\cos(6\pi \times 10^9 t + 30\pi x)\right] \quad \text{(mA/m)}.\end{aligned}$$

Since $k_1 = \omega\sqrt{\mu\varepsilon_1}$ and $k_2 = \omega\sqrt{\mu\varepsilon_2}$,

$$k_2 = \sqrt{\frac{\varepsilon_2}{\varepsilon_1}}\, k_1 = \sqrt{\frac{4}{2.25}}\, 30\pi = 40\pi \quad \text{(rad/m)},$$

$$\mathbf{E}_2 = \mathbf{E}^{\mathrm{t}} = \hat{\mathbf{y}}\, 8\tau \cos(6\pi \times 10^9 t - 40\pi x) = \hat{\mathbf{y}}\, 6.86 \cos(6\pi \times 10^9 t - 40\pi x) \quad \text{(V/m)},$$

$$\mathbf{H}_2 = \mathbf{H}^{\mathrm{t}} = \hat{\mathbf{z}}\, \frac{8\tau}{\eta_2} \cos(6\pi \times 10^9 t - 40\pi x) = \hat{\mathbf{z}}\, 36.38 \cos(6\pi \times 10^9 t - 40\pi x) \quad \text{(mA/m)}.$$

(b)

$$\begin{aligned}
\mathbf{S}^{\mathrm{i}}_{\mathrm{av}} &= \hat{\mathbf{x}}\, \frac{8^2}{2\eta_1} = \frac{64}{2 \times 251.33} = \hat{\mathbf{x}}\, 127.3 \quad (\text{mW/m}^2), \\
\mathbf{S}^{\mathrm{r}}_{\mathrm{av}} &= -|\Gamma|^2 \mathbf{S}^{\mathrm{i}}_{\mathrm{av}} = -\hat{\mathbf{x}}\, (0.143)^2 \times 0.127 = -\hat{\mathbf{x}}\, 2.6 \quad (\text{mW/m}^2), \\
\mathbf{S}^{\mathrm{t}}_{\mathrm{av}} &= \frac{|E^{\mathrm{t}}_0|^2}{2\eta_2} \\
&= \hat{\mathbf{x}}\, \tau^2 \frac{(8)^2}{2\eta_2} = \hat{\mathbf{x}}\, \frac{(0.86)^2 64}{2 \times 188.5} = \hat{\mathbf{x}}\, 124.7 \quad (\text{mW/m}^2).
\end{aligned}$$

Within calculation error, $\mathbf{S}^{\mathrm{i}}_{\mathrm{av}} + \mathbf{S}^{\mathrm{r}}_{\mathrm{av}} = \mathbf{S}^{\mathrm{t}}_{\mathrm{av}}$.

Problem 8.4 A 200-MHz, left-hand circularly polarized plane wave with an electric field modulus of 5 V/m is normally incident in air upon a dielectric medium with $\varepsilon_{\mathrm{r}} = 4$, and occupies the region defined by $z \geq 0$.

(a) Write an expression for the electric field phasor of the incident wave, given that the field is a positive maximum at $z = 0$ and $t = 0$.

(b) Calculate the reflection and transmission coefficients.

(c) Write expressions for the electric field phasors of the reflected wave, the transmitted wave, and the total field in the region $z \leq 0$.

(d) Determine the percentages of the incident average power reflected by the boundary and transmitted into the second medium.

Solution:

(a)

$$\begin{aligned}
k_1 &= \frac{\omega}{c} = \frac{2\pi \times 2 \times 10^8}{3 \times 10^8} = \frac{4\pi}{3} \text{ rad/m}, \\
k_2 &= \frac{\omega}{u_{\mathrm{p}_2}} = \frac{\omega}{c}\sqrt{\varepsilon_{\mathrm{r}_2}} = \frac{4\pi}{3}\sqrt{4} = \frac{8\pi}{3} \text{ rad/m}.
\end{aligned}$$

LHC wave:

$$\begin{aligned}
\widetilde{\mathbf{E}}^{\mathrm{i}} &= a_0(\hat{\mathbf{x}} + \hat{\mathbf{y}} e^{j\pi/2}) e^{-jkz} = a_0(\hat{\mathbf{x}} + j\hat{\mathbf{y}}) e^{-jkz}, \\
\mathbf{E}^{\mathrm{i}}(z,t) &= \hat{\mathbf{x}} a_0 \cos(\omega t - kz) - \hat{\mathbf{y}} a_0 \sin(\omega t - kz), \\
|\mathbf{E}^{\mathrm{i}}| &= [a_0^2 \cos^2(\omega t - kz) + a_0^2 \sin^2(\omega t - kz)]^{1/2} = a_0 = 5 \quad \text{(V/m)}.
\end{aligned}$$

Hence,

$$\widetilde{\mathbf{E}}^{\mathrm{i}} = 5(\hat{\mathbf{x}} + j\hat{\mathbf{y}}) e^{-j4\pi z/3} \quad \text{(V/m)}.$$

(b)

$$\eta_1 = \eta_0 = 120\pi \quad (\Omega), \qquad \eta_2 = \frac{\eta_0}{\sqrt{\varepsilon_{\mathrm{r}}}} = \frac{\eta_0}{2} = 60\pi \quad (\Omega).$$

Equations (8.8a) and (8.9) give

$$\Gamma = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1} = \frac{60\pi - 120\pi}{60\pi + 120\pi} = \frac{-60}{180} = -\frac{1}{3}, \qquad \tau = 1 + \Gamma = \frac{2}{3}.$$

(c)

$$\begin{aligned}
\widetilde{\mathbf{E}}^{\mathrm{r}} &= 5\Gamma(\hat{\mathbf{x}} + j\hat{\mathbf{y}}) e^{jk_1 z} = -\frac{5}{3}(\hat{\mathbf{x}} + j\hat{\mathbf{y}}) e^{j4\pi z/3} \quad \text{(V/m)}, \\
\widetilde{\mathbf{E}}^{\mathrm{t}} &= 5\tau(\hat{\mathbf{x}} + j\hat{\mathbf{y}}) e^{-jk_2 z} = \frac{10}{3}(\hat{\mathbf{x}} + j\hat{\mathbf{y}}) e^{-j8\pi z/3} \quad \text{(V/m)},
\end{aligned}$$

$$\widetilde{\mathbf{E}}_1 = \widetilde{\mathbf{E}}^{\mathrm{i}} + \widetilde{\mathbf{E}}^{\mathrm{r}} = 5(\hat{\mathbf{x}} + j\hat{\mathbf{y}}) \left[e^{-j4\pi z/3} - \frac{1}{3} e^{j4\pi z/3} \right] \quad \text{(V/m)}.$$

(d)

$$\%\text{ of reflected power} = 100 \times |\Gamma|^2 = \frac{100}{9} = 11.11\%,$$

$$\%\text{ of transmitted power} = 100 \times |\tau|^2 \frac{\eta_1}{\eta_2} = 100 \times \left(\frac{2}{3}\right)^2 \times \frac{120\pi}{60\pi} = 88.89\%.$$

Problem 8.10 For the configuration shown in Fig. P8.9, use transmission-line equations (or the Smith chart) to calculate the input impedance at $z = -d$ for $\varepsilon_{r_1} = 1$, $\varepsilon_{r_2} = 9$, $\varepsilon_{r_3} = 4$, $d = 1.2$ m, and $f = 50$ MHz. Also determine the fraction of the incident average power density reflected by the structure. Assume all media are lossless and nonmagnetic.

Solution: In medium 2,

$$\lambda = \frac{\lambda_0}{\sqrt{\varepsilon_{r_2}}} = \frac{c}{f\sqrt{\varepsilon_{r_2}}} = \frac{3 \times 10^8}{5 \times 10^7 \times 3} = 2 \text{ m}.$$

Hence,

$$\beta_2 = \frac{2\pi}{\lambda_2} = \pi \text{ rad/m}, \qquad \beta_2 d = 1.2\pi \text{ rad}.$$

At $z = -d$, the input impedance of a transmission line with load impedance Z_L is given by Eq. (2.63) as

$$Z_{\text{in}}(-d) = Z_0 \left(\frac{Z_L + jZ_0 \tan \beta_2 d}{Z_0 + jZ_L \tan \beta_2 d} \right).$$

In the present case, $Z_0 = \eta_2 = \eta_0/\sqrt{\varepsilon_{r_2}} = \eta_0/3$ and $Z_L = \eta_3 = \eta_0/\sqrt{\varepsilon_{r_3}} = \eta_0/2$, where $\eta_0 = 120\pi$ (Ω). Hence,

$$Z_{\text{in}}(-d) = \eta_2 \left(\frac{\eta_3 + j\eta_2 \tan \beta_2 d}{\eta_2 + j\eta_3 \tan \beta_2 d} \right) = \frac{\eta_0}{3} \left(\frac{\frac{1}{2} + j\left(\frac{1}{3}\right) \tan 1.2\pi}{\frac{1}{3} + j\left(\frac{1}{2}\right) \tan 1.2\pi} \right) = \eta_0 (0.35 - j0.14).$$

At $z = -d$,

$$\Gamma = \frac{Z_{\text{in}} - Z_1}{Z_{\text{in}} + Z_1} = \frac{\eta_0(0.35 - j0.14) - \eta_0}{\eta_0(0.35 - j0.14) + \eta_0} = 0.49 e^{-j162.14^\circ}.$$

Fraction of incident power reflected by the structure is $|\Gamma|^2 = |0.49|^2 = 0.24$.

Problem 8.16 A 0.5-MHz antenna carried by an airplane flying over the ocean surface generates a wave that approaches the water surface in the form of a normally incident plane wave with an electric-field amplitude of 3,000 (V/m). Seawater is characterized by $\varepsilon_r = 72$, $\mu_r = 1$, and $\sigma = 4$ (S/m). The plane is trying to communicate a message to a submarine submerged at a depth d below the water surface. If the submarine's receiver requires a minimum signal amplitude of 0.01 (μV/m), what is the maximum depth d to which successful communication is still possible?

Solution: For sea water at 0.5 MHz,

$$\frac{\varepsilon''}{\varepsilon'} = \frac{\sigma}{\omega\varepsilon} = \frac{4 \times 36\pi}{2\pi \times 0.5 \times 10^6 \times 72 \times 10^{-9}} = 2000.$$

Hence, sea water is a good conductor, in which case we use the following expressions from Table 7-1:

$$\begin{aligned}
\alpha_2 &= \sqrt{\pi f \mu \sigma} = \sqrt{\pi \times 0.5 \times 10^6 \times 4\pi \times 10^{-7} \times 4} = 2.81 \quad \text{(Np/m)},\\
\beta_2 &= 2.81 \quad \text{(rad/m)},\\
\eta_{c_2} &= (1+j)\frac{\alpha_2}{\sigma} = (1+j)\frac{2.81}{4} = 0.7\,(1+j)\ \Omega,\\
\Gamma &= \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1} = \frac{0.7\,(1+j) - 377}{0.7\,(1+j) + 377} = (-0.9963 + j3.7 \times 10^{-3}),\\
\tau &= 1 + \Gamma = 5.24 \times 10^{-3} e^{j44.89^\circ},\\
|E^{\rm t}| &= |\tau E_0^{\rm i} e^{-\alpha_2 d}|.
\end{aligned}$$

We need to find the depth z at which $|E^{\rm t}| = 0.01\ \mu$V/m $= 10^{-8}$ V/m.

$$\begin{aligned}
10^{-8} &= 5.24 \times 10^{-3} \times 3 \times 10^3 e^{-2.81\,d},\\
e^{-2.81\,d} &= 6.36 \times 10^{-10},\\
-2.81\,d &= \ln(6.36 \times 10^{-10}) = -21.18,
\end{aligned}$$

or

$$d = 7.54 \quad \text{(m)}.$$

Problem 8.20 A parallel-polarized plane wave is incident from air at an ar $\theta_i = 30°$ onto a pair of dielectric layers as shown in Fig. P8.20.

(a) Determine the angles of transmission θ_2, θ_3, and θ_4.

(b) Determine the lateral distance d.

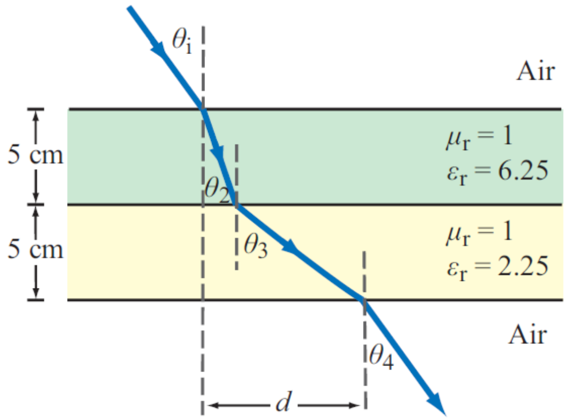


Figure P8.20: Problem P8.20.

Solution:

(a) Application of Snell's law of refraction given by (8.31) leads to:

$$\sin\theta_2 = \sin\theta_1\sqrt{\frac{\varepsilon_{r1}}{\varepsilon_{r2}}} = \sin 30°\sqrt{\frac{1}{6.25}} = 0.2$$
$$\theta_2 = 11.54°.$$

Similarly,

$$\sin\theta_3 = \sin\theta_2\sqrt{\frac{\varepsilon_{r2}}{\varepsilon_{r3}}} = \sin 11.54°\sqrt{\frac{6.25}{2.25}} = 0.33$$
$$\theta_3 = 19.48°.$$

And,

$$\sin\theta_4 = \sin\theta_3\sqrt{\frac{\varepsilon_{r3}}{\varepsilon_{r4}}} = \sin 19.48°\sqrt{\frac{2.25}{1}} = 0.5$$
$$\theta_4 = 30°.$$

As expected, the exit ray back into air will be at the same angle as θ_i.

(b)

$$\begin{aligned} d &= (5\text{ cm})\tan\theta_2 + (5\text{ cm})\tan\theta_3 \\ &= 5\tan 11.54° + 5\tan 19.48° = 2.79\text{ cm}. \end{aligned}$$

Problem 8.27 A plane wave in air with

$$\widetilde{\mathbf{E}}^{\mathrm{i}} = \hat{\mathbf{y}}\, 20e^{-j(3x+4z)} \quad \text{(V/m)}$$

is incident upon the planar surface of a dielectric material, with $\varepsilon_{\mathrm{r}} = 4$, occupying the half-space $z \geq 0$. Determine:

(a) The polarization of the incident wave.

(b) The angle of incidence.

(c) The time-domain expressions for the reflected electric and magnetic fields.

(d) The time-domain expressions for the transmitted electric and magnetic fields.

(e) The average power density carried by the wave in the dielectric medium.

Solution:

(a) $\widetilde{\mathbf{E}}^{\mathrm{i}} = \hat{\mathbf{y}}\, 20e^{-j(3x+4z)}$ V/m.

Since $\mathbf{E}^{\mathrm{i}}$ is along $\hat{\mathbf{y}}$, which is perpendicular to the plane of incidence, the wave is perpendicularly polarized.

(b) From Eq. (8.48a), the argument of the exponential is

$$-jk_1(x\sin\theta_{\mathrm{i}} + z\cos\theta_{\mathrm{i}}) = -j(3x+4z).$$

Hence,

$$k_1 \sin\theta_{\mathrm{i}} = 3, \qquad k_1 \cos\theta_{\mathrm{i}} = 4,$$

from which we determine that

$$\tan\theta_{\mathrm{i}} = \frac{3}{4} \qquad \text{or} \qquad \theta_{\mathrm{i}} = 36.87^\circ,$$

and

$$k_1 = \sqrt{3^2+4^2} = 5 \quad \text{(rad/m)}.$$

Also,

$$\omega = u_{\mathrm{p}} k = ck = 3\times 10^8 \times 5 = 1.5\times 10^9 \quad \text{(rad/s)}.$$

(c)

$$\eta_1 = \eta_0 = 377\ \Omega,$$

$$\eta_2 = \frac{\eta_0}{\sqrt{\varepsilon_{\mathrm{r}_2}}} = \frac{\eta_0}{2} = 188.5\ \Omega,$$

$$\theta_{\mathrm{t}} = \sin^{-1}\left[\frac{\sin\theta_{\mathrm{i}}}{\sqrt{\varepsilon_{\mathrm{r}_2}}}\right] = \sin^{-1}\left[\frac{\sin 36.87^\circ}{\sqrt{4}}\right] = 17.46^\circ,$$

$$\Gamma_\perp = \frac{\eta_2\cos\theta_{\mathrm{i}} - \eta_1\cos\theta_{\mathrm{t}}}{\eta_2\cos\theta_{\mathrm{i}} + \eta_1\cos\theta_{\mathrm{t}}} = -0.41,$$

$$\tau_\perp = 1 + \Gamma_\perp = 0.59.$$

In accordance with Eq. (8.49a), and using the relation $E_0^{\mathrm{r}} = \Gamma_\perp E_0^{\mathrm{i}}$,

$$\widetilde{\mathbf{E}}^{\mathrm{r}} = -\hat{\mathbf{y}}\, 8.2\, e^{-j(3x-4z)},$$
$$\widetilde{\mathbf{H}}^{\mathrm{r}} = -(\hat{\mathbf{x}} \cos\theta_{\mathrm{i}} + \hat{\mathbf{z}} \sin\theta_{\mathrm{i}})\, \frac{8.2}{\eta_0}\, e^{-j(3x-4z)},$$

where we used the fact that $\theta_{\mathrm{i}} = \theta_{\mathrm{r}}$ and the z-direction has been reversed.

$$\mathbf{E}^{\mathrm{r}} = \mathfrak{Re}[\widetilde{\mathbf{E}}^{\mathrm{r}} e^{j\omega t}] = -\hat{\mathbf{y}}\, 8.2 \cos(1.5 \times 10^9 t - 3x + 4z) \quad \text{(V/m)},$$
$$\mathbf{H}^{\mathrm{r}} = -(\hat{\mathbf{x}}\, 17.4 + \hat{\mathbf{z}}\, 13.06) \cos(1.5 \times 10^9 t - 3x + 4z) \quad \text{(mA/m)}.$$

(d) In medium 2,

$$k_2 = k_1 \sqrt{\frac{\varepsilon_2}{\varepsilon_1}} = 5\sqrt{4} = 20 \quad \text{(rad/m)},$$

and

$$\theta_{\mathrm{t}} = \sin^{-1}\left[\sqrt{\frac{\varepsilon_1}{\varepsilon_2}} \sin\theta_{\mathrm{i}}\right] = \sin^{-1}\left[\frac{1}{2} \sin 36.87^\circ\right] = 17.46^\circ$$

and the exponent of $\mathbf{E}^{\mathrm{t}}$ and $\mathbf{H}^{\mathrm{t}}$ is

$$-jk_2(x \sin\theta_{\mathrm{t}} + z \cos\theta_{\mathrm{t}}) = -j10(x \sin 17.46^\circ + z \cos 17.46^\circ) = -j(3x + 9.54z).$$

Hence,

$$\widetilde{\mathbf{E}}^{\mathrm{t}} = \hat{\mathbf{y}}\, 20 \times 0.59\, e^{-j(3x+9.54z)},$$
$$\widetilde{\mathbf{H}}^{\mathrm{t}} = (-\hat{\mathbf{x}} \cos\theta_{\mathrm{t}} + \hat{\mathbf{z}} \sin\theta_{\mathrm{t}})\, \frac{20 \times 0.59}{\eta_2}\, e^{-j(3x+9.54z)}.$$
$$\mathbf{E}^{\mathrm{t}} = \mathfrak{Re}[\widetilde{\mathbf{E}}^{\mathrm{t}} e^{j\omega t}] = \hat{\mathbf{y}}\, 11.8 \cos(1.5 \times 10^9 t - 3x - 9.54z) \quad \text{(V/m)},$$
$$\mathbf{H}^{\mathrm{t}} = (-\hat{\mathbf{x}} \cos 17.46^\circ + \hat{\mathbf{z}} \sin 17.46^\circ)\, \frac{11.8}{188.5} \cos(1.5 \times 10^9 t - 3x - 9.54z)$$
$$= (-\hat{\mathbf{x}}\, 59.72 + \hat{\mathbf{z}}\, 18.78) \cos(1.5 \times 10^9 t - 3x - 9.54z) \quad \text{(mA/m)}.$$

(e)

$$S_{\mathrm{av}}^{\mathrm{t}} = \frac{|E_0^{\mathrm{t}}|^2}{2\eta_2} = \frac{(11.8)^2}{2 \times 188.5} = 0.36 \quad \text{(W/m}^2\text{)}.$$

Problem 8.29 A plane wave in air with

$$\widetilde{\mathbf{E}}^{\mathrm{i}} = (\hat{\mathbf{x}}9 - \hat{\mathbf{y}}4 - \hat{\mathbf{z}}6)e^{-j(2x+3z)} \quad \text{(V/m)}$$

is incident upon the planar surface of a dielectric material, with $\varepsilon_{\mathrm{r}} = 2.25$, occupying the half-space $z \geq 0$. Determine

(a) The incidence angle θ_{i}.

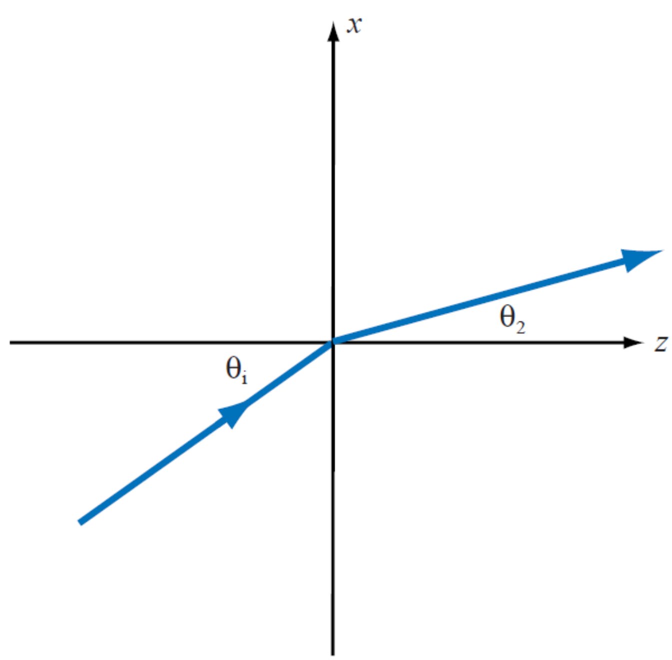
(b) The frequency of the wave.

(c) The field $\widetilde{\mathbf{E}}^{\mathrm{r}}$ of the reflected wave.

(d) The field $\widetilde{\mathbf{E}}^{\mathrm{t}}$ of the wave transmitted into the dielectric medium.

(e) The average power density carried by the wave into the dielectric medium.

Solution:



(a) From the exponential of the given expression, it is clear that the wave direction of travel is in the x–z plane. By comparison with the expressions in (8.48a) for perpendicular polarization or (8.65a) for parallel polarization, both of which have the same phase factor, we conclude that:

$$k_1 \sin\theta_{\mathrm{i}} = 2,$$
$$k_1 \cos\theta_{\mathrm{i}} = 3.$$

Hence,

$$k_1 = \sqrt{2^2 + 3^2} = 3.6 \quad \text{(rad/m)}$$
$$\theta_i = \tan^{-1}(2/3) = 33.7^\circ.$$

Also,

$$k_2 = k_1\sqrt{\varepsilon_{r_2}} = 3.6\sqrt{2.25} = 5.4 \quad \text{(rad/m)}$$
$$\theta_2 = \sin^{-1}\left[\sin\theta_i\sqrt{\frac{1}{2.25}}\right] = 21.7^\circ.$$

(b)

$$k_1 = \frac{2\pi f}{c}$$
$$f = \frac{k_1 c}{2\pi} = \frac{3.6 \times 3 \times 10^8}{2\pi} = 172 \text{ MHz}.$$

(c) In order to determine the electric field of the reflected wave, we first have to determine the polarization of the wave. The vector argument in the given expression for $\widetilde{\mathbf{E}}^i$ indicates that the incident wave is a mixture of parallel and perpendicular polarization components. Perpendicular polarization has a $\hat{\mathbf{y}}$-component only (see 8.46a), whereas parallel polarization has only $\hat{\mathbf{x}}$ and $\hat{\mathbf{z}}$ components (see 8.65a). Hence, we shall decompose the incident wave accordingly:

$$\widetilde{\mathbf{E}}^i = \widetilde{\mathbf{E}}^i_\perp + \widetilde{\mathbf{E}}^i_\parallel$$

with

$$\widetilde{\mathbf{E}}^i_\perp = -\hat{\mathbf{y}}\,4e^{-j(2x+3z)} \quad \text{(V/m)}$$
$$\widetilde{\mathbf{E}}^i_\parallel = (\hat{\mathbf{x}}9 - \hat{\mathbf{z}}6)e^{-j(2x+3z)} \quad \text{(V/m)}$$

From the above expressions, we deduce:

$$E^i_{\perp 0} = -4 \text{ V/m}$$
$$E^i_{\parallel 0} = \sqrt{9^2 + 6^2} = 10.82 \text{ V/m}.$$

Next, we calculate Γ and τ for each of the two polarizations:

$$\Gamma_\perp = \frac{\cos\theta_i - \sqrt{(\varepsilon_2/\varepsilon_1) - \sin^2\theta_i}}{\cos\theta_i + \sqrt{(\varepsilon_2/\varepsilon_1) - \sin^2\theta_i}}$$

Using $\theta_\text{i} = 33.7^\circ$ and $\varepsilon_2/\varepsilon_1 = 2.25/1 = 2.25$ leads to:

$$\begin{aligned}
\Gamma_\perp &= -0.25 \\
\tau_\perp &= 1 + \Gamma_\perp = 0.75.
\end{aligned}$$

Similarly,

$$\begin{aligned}
\Gamma_\perp &= \frac{-(\varepsilon_2/\varepsilon_1)\cos\theta_\text{i} + \sqrt{(\varepsilon_2/\varepsilon_1) - \sin^2\theta_\text{i}}}{(\varepsilon_2/\varepsilon_1)\cos\theta_\text{i} + \sqrt{(\varepsilon_2/\varepsilon_1) - \sin^2\theta_\text{i}}} = -0.15, \\
\tau_\parallel &= (1 + \Gamma_\parallel)\frac{\cos\theta_\text{i}}{\cos\theta_\text{t}} = (1 - 0.15)\frac{\cos 33.7^\circ}{\cos 21.7^\circ} = 0.76.
\end{aligned}$$

The electric fields of the reflected and transmitted waves for the two polarizations are given by (8.49a), (8.49c), (8.65c), and (8.65e):

$$\begin{aligned}
\widetilde{\mathbf{E}}_\perp^\text{r} &= \hat{\mathbf{y}} E_{\perp 0}^\text{r} e^{-jk_1(x\sin\theta_\text{r} - z\cos\theta_\text{r})} \\
\widetilde{\mathbf{E}}_\perp^\text{t} &= \hat{\mathbf{y}} E_{\perp 0}^\text{t} e^{-jk_2(x\sin\theta_\text{t} + z\cos\theta_\text{t})} \\
\widetilde{\mathbf{E}}_\parallel^\text{r} &= (\hat{\mathbf{x}}\cos\theta_\text{r} + \hat{\mathbf{z}}\sin\theta_\text{r}) E_{\parallel 0}^\text{r} e^{-jk_1(x\sin\theta_\text{r} - z\cos\theta_\text{r})} \\
\widetilde{\mathbf{E}}_\parallel^\text{t} &= (\hat{\mathbf{x}}\cos\theta_\text{t} - \hat{\mathbf{z}}\sin\theta_\text{t}) E_{\parallel 0}^\text{t} e^{-jk_2(x\sin\theta_\text{t} + z\cos\theta_\text{t})}
\end{aligned}$$

Based on our earlier calculations:

$$\begin{aligned}
\theta_\text{r} &= \theta_\text{i} = 33.7^\circ \\
\theta_\text{t} &= 21.7^\circ \\
k_1 &= 3.6 \text{ rad/m}, \qquad k_2 = 5.4 \text{ rad/m}, \\
E_{\perp 0}^\text{r} &= \Gamma_\perp E_{\perp 0}^\text{i} = (-0.25) \times (-4) = 1 \text{ V/m}. \\
E_{\perp 0}^\text{t} &= \tau_\perp E_{\perp 0}^\text{i} = 0.75 \times (-4) = -3 \text{ V/m}. \\
E_{\parallel 0}^\text{r} &= \Gamma_\parallel E_{\parallel 0}^\text{i} = (-0.15) \times 10.82 = -1.62 \text{ V/m}. \\
E_{\parallel 0}^\text{t} &= \tau_\parallel E_{\parallel 0}^\text{i} = 0.76 \times 10.82 = 8.22 \text{ V/m}.
\end{aligned}$$

Using the above values, we have:

$$\begin{aligned}
\widetilde{\mathbf{E}}^\text{r} &= \widetilde{\mathbf{E}}_\perp^\text{r} + \widetilde{\mathbf{E}}_\parallel^\text{r} \\
&= (\hat{\mathbf{x}} E_{\parallel 0}^\text{r}\cos\theta_\text{r} + \hat{\mathbf{y}} E_{\perp 0}^\text{r} + \hat{\mathbf{z}} E_{\parallel 0}^\text{r}\sin\theta_\text{r}) e^{-j(2x - 3z)} \\
&= (-\hat{\mathbf{x}}\,1.35 + \hat{\mathbf{y}} - \hat{\mathbf{z}}\,0.90) e^{-j(2x - 3z)} \quad \text{(V/m)}.
\end{aligned}$$

(d)

$$\begin{aligned}\widetilde{\mathbf{E}}^{\mathrm{t}} &= \widetilde{\mathbf{E}}^{\mathrm{t}}_{\perp} + \widetilde{\mathbf{E}}^{\mathrm{t}}_{\parallel} \\ &= (\hat{\mathbf{x}}7.65 - \hat{\mathbf{y}}3 - \hat{\mathbf{z}}3.05)e^{-j(2x+5z)} \quad \text{(V/m)}.\end{aligned}$$

(e)

$$\begin{aligned}S^{\mathrm{t}} &= \frac{|E^{\mathrm{t}}_0|^2}{2\eta_2} \\ |E^{\mathrm{t}}_0|^2 &= (7.65)^2 + 3^2 + (3.05)^2 = 76.83 \\ \eta_2 &= \frac{\eta_0}{\sqrt{\varepsilon_{\mathrm{r}_2}}} = \frac{377}{1.5} = 251.3\ \Omega \\ S^{\mathrm{t}} &= \frac{76.83}{2 \times 251.3} = 152.86 \quad (\text{mW/m}^2).\end{aligned}$$

Problem 8.35 A parallel-polarized beam of light with an electric field amplitude of 10 (V/m) is incident in air on polystyrene with $\mu_{\mathrm{r}} = 1$ and $\varepsilon_{\mathrm{r}} = 2.6$. If the incidence angle at the air–polystyrene planar boundary is 50°, determine the following:

(a) The reflectivity and transmissivity.

(b) The power carried by the incident, reflected, and transmitted beams if the spot on the boundary illuminated by the incident beam is 1 m^2 in area.

Solution:

(a) From Eq. (8.68),

$$\begin{aligned}\Gamma_{\parallel} &= \frac{-(\varepsilon_2/\varepsilon_1)\cos\theta_{\mathrm{i}} + \sqrt{(\varepsilon_2/\varepsilon_1) - \sin^2\theta_{\mathrm{i}}}}{(\varepsilon_2/\varepsilon_1)\cos\theta_{\mathrm{i}} + \sqrt{(\varepsilon_2/\varepsilon_1) - \sin^2\theta_{\mathrm{i}}}} \\ &= \frac{-2.6\cos 50^\circ + \sqrt{2.6 - \sin^2 50^\circ}}{2.6\cos 50^\circ + \sqrt{2.6 - \sin^2 50^\circ}} = -0.08, \\ R_{\parallel} &= |\Gamma_{\parallel}|^2 = (0.08)^2 = 6.4 \times 10^{-3}, \\ T_{\parallel} &= 1 - R_{\parallel} = 0.9936.\end{aligned}$$

(b)

$$\begin{aligned}P^{\mathrm{i}}_{\parallel} &= \frac{|E^{\mathrm{i}}_{\parallel 0}|^2}{2\eta_1} A\cos\theta_{\mathrm{i}} = \frac{(10)^2}{2 \times 120\pi} \times \cos 50^\circ = 85\ \text{mW}, \\ P^{\mathrm{r}}_{\parallel} &= R_{\parallel} P^{\mathrm{i}}_{\parallel} = (6.4 \times 10^{-3}) \times 0.085 = 0.55\ \text{mW}, \\ P^{\mathrm{t}}_{\parallel} &= T_{\parallel} P^{\mathrm{i}}_{\parallel} = 0.9936 \times 0.085 = 84.45\ \text{mW}.\end{aligned}$$

Problem 8.39 A hollow rectangular waveguide is to be used to transmit signals at a carrier frequency of 6 GHz. Choose its dimensions so that the cutoff frequency of the dominant TE mode is lower than the carrier by 25% and that of the next mode is at least 25% higher than the carrier.

Solution:

For $m = 1$ and $n = 0$ (TE$_{10}$ mode) and $u_{p_0} = c$ (hollow guide), Eq. (8.106) reduces to

$$f_{10} = \frac{c}{2a}\,.$$

Denote the carrier frequency as $f_0 = 6$ GHz. Setting

$$f_{10} = 0.75 f_0 = 0.75 \times 6 \text{ GHz} = 4.5 \text{ GHz},$$

we have

$$a = \frac{c}{2f_{10}} = \frac{3 \times 10^8}{2 \times 4.5 \times 10^9} = 3.33 \text{ cm}.$$

If b is chosen such that $a > b > \frac{a}{2}$, the second mode will be TE$_{01}$, followed by TE$_{20}$ at $f_{20} = 9$ GHz. For TE$_{01}$,

$$f_{01} = \frac{c}{2b}\,.$$

Setting $f_{01} = 1.25 f_0 = 7.5$ GHz, we get

$$b = \frac{c}{2f_{01}} = \frac{3 \times 10^8}{2 \times 7.5 \times 10^9} = 2 \text{ cm}.$$

Problem 8.41 A waveguide filled with a material whose $\varepsilon_r = 2.25$ has dimensions $a = 2$ cm and $b = 1.4$ cm. If the guide is to transmit 10.5-GHz signals, what possible modes can be used for the transmission?

Solution:

Application of Eq. (8.106) with $u_{p_0} = c/\sqrt{\varepsilon_r} = 3 \times 10^8/\sqrt{2.25} = 2 \times 10^8$ m/s, gives:

$$\begin{aligned}
f_{10} &= 5 \text{ GHz (TE only)}\\
f_{01} &= 7.14 \text{ GHz (TE only)}\\
f_{11} &= 8.72 \text{ GHz (TE or TM)}\\
f_{20} &= 10 \text{ GHz (TE only)}\\
f_{21} &= 12.28 \text{ GHz (TE or TM)}\\
f_{12} &= 15.1 \text{ GHz (TE or TM)}.
\end{aligned}$$

Hence, any one of the first four modes can be used to transmit 10.5-GHz signals.

Problem 8.47 A hollow cavity made of aluminum has dimensions $a = 4$ cm and $d = 3$ cm. Calculate Q of the TE_{101} mode for

(a) $b = 2$ cm, and

(b) $b = 3$ cm.

Solution:

For the TE_{101} mode, f_{101} is independent of b,

$$\begin{aligned} f_{101} &= \frac{c}{2}\sqrt{\left(\frac{1}{a}\right)^2 + \left(\frac{1}{d}\right)^2} \\ &= \frac{3 \times 10^8}{2}\sqrt{\left(\frac{1}{4 \times 10^{-2}}\right)^2 + \left(\frac{1}{3 \times 10^{-2}}\right)^2} \\ &= 6.25 \text{ GHz}. \end{aligned}$$

For aluminum with $\sigma_c = 3.5 \times 10^7$ S/m (Appendix B),

$$\delta_s = \frac{1}{\sqrt{\pi f_{101} \mu_0 \sigma_c}} = 1.08 \times 10^{-6} \text{ m}.$$

(a) For $a = 4$ cm, $b = 2$ cm and $d = 3$ cm,

$$\begin{aligned} Q &= \frac{1}{\delta_s} \frac{abd(a^2 + d^2)}{[a^3(d + 2b) + d^3(a + 2b)]} \\ &= 8367. \end{aligned}$$

(b) For $a = 4$ cm, $b = 3$ cm, and $d = 3$ cm,

$$Q = 9850.$$